Nim

UF Math Circle Activity

Brief History of Nim

Nim is an ancient game, believed to have originated in China. The name "Nim" was given by Charles L. Bouton, a Harvard professor, in 1901. Despite its simple rules, Nim has a deep mathematical theory that was fully developed by Bouton.

Rules of the Game

- Start with three piles of objects (e.g., 3, 5, and 7 objects in each pile).
- Two players take turns.
- On your turn, you must take any number of objects from exactly one pile.
- The player who takes the last object wins.

Exploring Winning Strategies

Work with a partner or in a small group to explore the following questions. Write down your thoughts and strategies.

Basic Questions

- Can you find a strategy that always lets you win? Try playing several rounds with different starting piles.
- What happens if you leave your opponent with two equal piles?
- What happens if you leave your opponent with one pile empty and the other two piles equal?

Binary Representation

Nim involves more than just removing objects. There's a hidden pattern based on the binary (base 2) representation of the numbers. Try the following:

- Convert the number of objects in each pile into binary (e.g., 5 objects is 101 in binary).

- Line up the binary numbers for each pile and add the digits in each column.
- What do you notice when the sum in each column is 0 or 2?
- Can you find a way to use this to decide your moves?

Deriving the Key Theorem

Challenge: Prove or disprove the following statement: If you can leave a situation where the sum in each binary column is 0, you can always win.

- Start by exploring different positions and converting them to binary.
- Try to find a pattern or rule that helps you win from these positions.
- Can you formalize your findings into a theorem?

Extension: Generalizing Nim

For those looking for a deeper challenge, consider the following variations:

- What happens if there are more than three piles? Does your strategy still work?
- Modify the game so that the player who takes the last object loses. How does this change your strategy?
- Explore the probability of starting with a "safe" combination (where the sum of each binary column is 0 or 2). What fraction of possible starting positions are safe?

Reflection

After exploring the questions above, take some time to reflect:

- How did your understanding of binary numbers help you with Nim?
- What surprised you about the game?